HADRONIC ELECTROMAGNETIC FORM FACTORS AND COLOR TRANSPARENCY

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We review the current status of electromagnetic form factor calculations in perturbative QCD. There is growing evidence that factorization prescriptions involving a transverse coordinate integration, such as that of Li and Sterman, are more appropriate than the prescription of LePage and Brodsky. Color transparency is naturally described within the formalism. We report the first explicit calculations of color transparency and nuclear filtering as perturbatively calculable phenomena.

1 Introduction

The applicability of perturbative QCD to exclusive processes has always been controversial. Despite the remarkable agreement with data of the quark-counting scaling laws of Brodsky and Farrar, the helicity conservation selection rules of Lepage and Brodsky tend not to agree with data [1,2,3].

It has not been clear how to interpret this conflict. By dimensional analysis, scaling indicates that a finite, minimal number of quarks is being probed. However, the failure of hadronic helicity conservation is a direct test of the factorization scheme, and the failure cannot be repaired by appealing to models of distribution amplitudes or their normalizations. Failure of hadron helicity conservation apparently rules out dominance by the short distance formalism. It has been common to identify the short-distance formulation as being "the same as" perturbative QCD ($pQCD$) itself. Then the agreement of the scaling laws with experiments appears to be rather mysterious.

Theoretical criticisms focus on calculations found to include regions where the internal momentum transfers are too small for leading order $pQCD$ to apply reliably [4,5]. For even the simplest model calculations, the case of hadronic form factors, it is found that large contributions come from the components of quark wave functions involving large quark spatial separations.

A reasonable resolution of the conflict observes that a factorization scheme is merely a tool, in which different amplitudes are re-arranged for the purpose

of calculation. Hence if one factorization scheme is inapplicable to experiment, one can always try another, and the underlying applicability of the approximations may be improved. Li and Sterman [6,7] gave an improved factorization formula for calculating the pion and the proton form factor, which included Sudakov suppression. The Sudakov form factor tends to suppress the regions of large quark spatial separations, thereby extending the applicability of $pQCD$. We will review the mechanism and our calculations in detail below.

Unfortunately the Sudakov effect is not sufficiently dramatic to resolve all the issues in the region of current momentum transfers (Q^2). This provides an additional strong motivation for extending the experimental scope, and in particular for studying quasi-exclusive reactions in nuclear targets.

The early conception of color transparency [8] was based on having large momentum transfer Q^2 select short distance regions, freed with color coherence to propagate through a passive nuclear probe. Given the controversies over short-distance dominance, it has not been clear whether large enough Q^2 would be obtained to make the basic assumptions apply at laboratory energies. However, large quark separations should tend to be absorbed in the strongly interacting nuclear medium, while small quark separations should penetrate freely[9,10,11]. This is the phenomenon called "nuclear filtering", which acts somewhat like Sudakov suppression. Instead of Q^2 as the large dimensionful scale, there is the large nuclear radius of order $A^{1/3} fm$.

Both Sudakov effects and nuclear filtering depend directly on the transverse coordinate. In fact, the transverse-position space factorization in which color transparency and nuclear filtering is described [10] pre-dates the very similar factorization of Li and Sterman [6,7]. Both hearken to the proton-proton scattering work of Botts and Sterman [12], which was constructed to address the inability of Lepage-Brodsky factorization to describe independent scattering. Because all of the ideas spring from a common factorization prescription, the explicit calculations dovetail together perfectly, and they can be presented in the same format.

2   Hadronic Form Factors

LePage and Brodsky [2] calculate the pion electromagnetic form factor at momentum transfer $q^2 = -Q^2$ with a factorization method written as

$$F_\pi(Q^2) = \int dx_1 dx_2 \phi(x_2, Q) H(x_1, x_2, Q) \phi(x_1, Q). \tag{1}$$

Here $\phi(x, Q)$ are the distribution amplitudes, which can be expressed in terms of the pion wave function $\psi(x, \vec{k}_T)$ as

$$\phi(x, Q) = \int^Q d^2 k_T \psi(x, \vec{k}_T). \qquad (2)$$

We use x for the longitudinal momentum fraction and $\vec{k}_T$ for the transverse momentum carried by the quark. The factorization is justified provided the external photon momentum Q^2 is *asymptotically large*. Then the k_T integrals decouple, and the k_T dependence of the hard scattering H can be expanded in a power series, retaining the trivial, constant term. One directly obtains the power-law scaling of the quark-counting method, with logarithmic corrections.

Unfortunately, several authors [4,5] have found that much of the numerical weight of explicit calculations comes from the end point regions. The proposed decoupling of the transverse integrations is not a numerically accurate approximation, making application of the method suspect. We reiterate that this legitimate source of doubt has often been extended to the whole application of $pQCD$ [4,5] to hard exclusive processes. Since $pQCD$ and the factorization scheme are separate concepts, it is unjustified to jump to such a conclusion.

The Li-Sterman factorization [6,12] retains coupling of the k_T dependence of the wave functions and the hard scattering. In some sense the concept is less ambitious theoretically, by including a broader integration region than the zero-distance LePage-Brodsky method. The calculation is simplified by dropping the weak k_T dependence of quark propagators in a hard scattering kernel H. No loss of consistency occurs, because the rest of the k_T dependence is sufficient to justify this step. Working in configuration space [6] the usual convolutions become a product:

$$F_\pi(Q^2) = \int dx_1 dx_2 \frac{d^2\vec{b}}{(2\pi)^2} \mathcal{P}(x_2, b, P_2, \mu) \tilde{H}(x_1, x_2, Q^2, \vec{b}, \mu) \mathcal{P}(x_1, b, P_1, \mu), \quad (3)$$

where $\mathcal{P}(x, b, P, \mu)$ and $\tilde{H}(x_1, x_2, Q^2, \vec{b}, \mu)$ are the Fourier transforms of the wave function, including Sudakov factors, and hard scattering respectively; $\vec{b}$ is conjugate to $\vec{k}_{T1} - \vec{k}_{T2}$, μ is the renormalization scale and P_1, P_2 are the initial and final momenta of the pion.

The leading-order Li-Sterman method is marginally consistent in practice. If the dependence on a transverse separation cutoff [6] is studied at Q^2 of a few GeV^2, then nearly 50% of the form factor comes from a region where $\alpha_s/\pi < 0.7$. This indicates that higher order contributions in α_s may not be negligible, but this is a separate issue from the factorization scheme. Certainly the leading order predictions for the normalization of the form factor cannot be regarded

as accurate. Next to leading order calculation[14] of the pion form factor confirm this conclusion. Numerically, the fact that results of the calculations (described in detail elsewhere [32]) lie below the experimental data cannot be given great weight; in fact, the agreement is actually quite acceptable.

2.1 The Proton

The proton Dirac form factor $F_1^p(Q^2)^7$ is considerably more complicated. In contrast to the pion, there is no natural choice for the infrared cutoff in the Sudakov exponent, due to the presence of three quarks and resulting three distances.

Bolz et. al[15] pointed out that the infra-red cut-off b_c used by Li[7] does not suppress the soft divergences as $b_c \to 1/\Lambda_{QCD}$. A modified choice of the cutoffs was proposed by them [15]. Subsequently the form factor was found to saturate as $b_c \to 1/\Lambda_{QCD}$. The normalization of the resulting $Q^4 F_1$ was found to be less than half of that of the data for all the distribution amplitudes explored [15]. Bolz et al[15] then concluded that $pQCD$ is unable to fit the experimental form factor.

Kundu et al[16] re-examined the situation. Considerably more complete calculations were performed[16], incorporating the full two-loop correction to the Sudakov effects. A physical choice of the infra-red cutoff parameter was also incorporated. This cut-off prescription treats the proton as a quark-diquark configuration at the extreme point of quark separation.

As a result, Kundu et al[16] find that the calculation is in good agreement with data using the King-Sachrajda (KS)[17] distribution amplitude. The fact that the normalization of the proton form factor can be fit makes an important conceptual point: the method in principle can explain the data, if higher order corrections in α_s were under control. Again this is supported by examination of the contributing integration regions, or b_c dependence[16]. Saturation occurs at about $b_c = 0.8/\Lambda_{QCD}$, so about 50% of the calculation comes from the soft or the large b regions. Scaling is postponed to beyond $Q^2 = 10 GeV^2$, but is inevitable after that.

The pion and the proton form factor calculations reveal that short-distance regions, required to be dominant in the basic LePage-Brodsky factorization, do not dominate in practice. Other prescriptions (such as Li and Sterman) are capable of incorporating long-distance regions. The Q^2 scaling dependence of the proton form factor above about $10 GeV^2$ appears to be quite robust. The calculations are sufficiently independent of the theoretical uncertainties such as distribution amplitude models, the infra-red cut-off parameters, and higher-order corrections, to indicate that current large Q^2 experimental scaling

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is truly fundamental.

3 Color Transparency and Nuclear Filtering

Color transparency is a natural prediction of $pQCD$. However, if the asymptotic limit is taken prematurely (as in LePage-Brodsky factorization) then all targets have perfect transparency, and there is nothing left to calculate. Taking Q^2 indefinitely large (but fixed), one might think all targets become transparent. But then taking $A \to \infty$ all targets become opaque. Thus there is a limit interchange problem in the LePage-Brodsky factorization, because the limit of large Q^2 and large A do not commute. The scheme is limited to asymptotic Q^2, and fundamentally unable to describe the phenomena at laboratory energies.

A correct description of the phenomenon follows from a factorization scheme incorporating the transverse degrees of freedom [10]. It is very useful that we do not have to rely on extremely large Q^2 to motivate $pQCD$, but instead large A serves as an infrared cutoff. The "filtering limit" takes $A >> 1$ with Q^2 fixed and large enough to motivate a $pQCD$ approach to attenuation: this requires merely $Q^2 >$ a few GeV^2. On this basis it has been predicted that perturbative QCD calculations are more reliable in a nuclear target.

These concepts have experimental support. Experimentally one finds that the fixed-angle free space process $pp' \to p''p'''$ [19] shows significant oscillations at 90 degrees as a function of energy. The energy region of oscillations extends over the whole range of high energy measurements that exist, from $s = 6GeV^2$ to $s = 40GeV^2$. The oscillations are large, making up roughly 50% of the $1/s^{10}$ behavior, and are interpreted as coming from interference of long- and short-distance amplitudes. The corresponding process in a nuclear environment $pA \to p'p''(A-1)$ shows no oscillations, and obeys the $pQCD$ scaling power law far better than the free-space data[9,20,11]. The A dependence, when analyzed at fixed Q^2, shows statistically significant evidence of reduced attenuation[21].

While the formalism and model calculations have existed for a while, the calculations to verify it are quite complex. Only with the completion of the work by Li et. al [6] were all the pieces to make the complete perturbative calculation laid out in ordered form. The calculations require Monte Carlo integrations of very high order (up to 9 dimensions after taking into account all symmetries) which have never before been attempted. The results, however, are encouraging and show that the nuclear interactions do substantially eliminate the soft region [22].
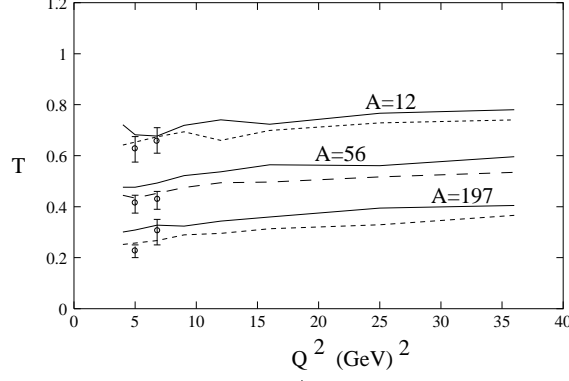
Figure 1: The calculated transparency ratio for the proton for different nuclei. The experimental points are taken from Ref. [29,30]. The solid curves are calculated with $k = 5$ and the dashed curves with $k = 6$.

3.1 The Pion: Nuclear Medium Effects

The nuclear medium modifies the quark wave function such that [10]

$$\mathcal{P}_A(x, b, P, \mu) = f_A(b; B)\mathcal{P}(x, b, P, \mu), \tag{4}$$

where $\mathcal{P}_A$ is the wave function inside the medium and f_A is the nuclear filtering amplitude. An eikonal form [23,24] appropriate for f_A is:

$$f_A(b; B) = exp(-\int_z^\infty dz'\sigma(b)\rho(B, z')/2). \tag{5}$$

Here $\rho(B, z')$ is the nuclear number density [27] at longitudinal distance z and impact parameter B relative to the nuclear center. We have used the fact that the imaginary part of the eikonal amplitude for forward scattering is related to the total cross section, explaining our use of the symbol $\sigma(b)/2$. Finally, we must include the probability to find a pion at position B, z inside the nucleus, which we take to be a constant times the probability to find a nucleon. Putting together the factors, the process of knocking out a pion inside a nuclear target has an amplitude M given by

$$M = \int_0^\infty d^2B \int_{-\infty}^{+\infty} dz\rho(B, z) \times F_\pi(x_1, x_2, b, Q^2) \times f_A(b, B) \tag{6}$$
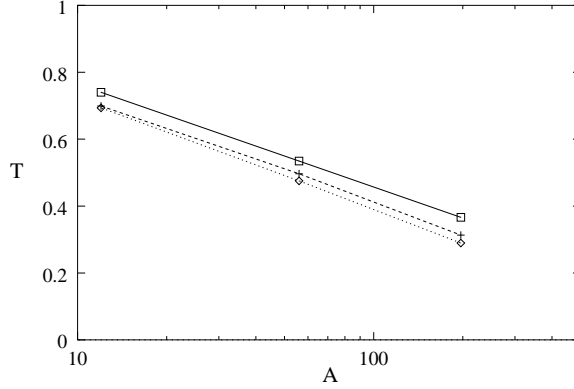
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Figure 2: The calculated transparency ratio for the proton as a function of A for different Q^2. The solid, dashed and dotted curves correspond to $Q^2 = 36$, 16 and 9 GeV^2 respectively. The value of the parameter $k = 6$.

The inelastic cross section σ is known to scale like b^2 in QCD [25,26]. We parametrize $\sigma(b)$ as kb^2 and adjust the value of k to find a reasonable fit to the experimental data.

3.2 The Proton: Nuclear Medium Effects

For the proton the important transverse scale is the maximum of the three quark separation distances, $b_{max} = max(b_1, b_2, b_3)$. The calculation of the process in the nuclear target needs a 9 dimensional integration, which is performed by Monte Carlo. The effects of short-range correlations were included approximately by replacing [28]

$$\rho(z', b) \rightarrow \rho(z', b)C(|z - z'|), \tag{7}$$

where $C(u)$ is a correlation function estimated in [29] to be $C(u) = [g(u)]^{1/2}$ with

$$g(u) = \left[1 - \frac{h(u)^2}{4}\right][1 + f(u)]^2 \tag{8}$$

where

$$h(u) = 3\frac{j_1(k_F u)}{k_F u}, \tag{9}$$

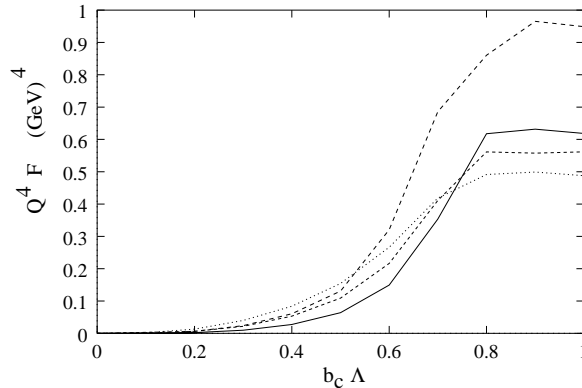$$f(u) = -e^{-\alpha u^2}(1 - \beta u^2) \tag{10}$$

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Figure 3: The transverse separation cutoff b_c dependence of the proton amplitude ratio. Curves are drawn for $A = 197$. The solid, short dashed and the dotted curves are calculated for $Q^2 = 6.8$, 16 and 36 GeV^2 respectively. The long dashed curve corresponds to the free space calculation for $Q^2 = 16$ GeV^2, which contains substantially more long-distance contamination.

with $\alpha = 1.1$, $\beta = 0.68$ fm^{-2} and the Fermi momentum $k_F = 1.36$ fm^{-1}.

4 Results and Discussions

Results for the Q^2 dependence of the proton transparency ratio are given in Fig. 1 and 2. The parameter k in the attenuation cross section $\sigma = kb^2$ was chosen so as to provide a reasonable fit to the experimental data [30,31].

The quark transverse separation cutoff b_c dependence of the amplitude ratio is shown in Fig. 3. It is clear from this figure that the large distance contributions are significantly reduced in the nuclear environment. We find that for a heavy nucleus at 36 GeV^2, 90% of the contribution comes from a region where α_s/π is less than 0.7.

We have also checked the dependence of our result on the infrared cutoff parameter c and the choice of the wave function. We find that the results for transparency ratio change very little if we use the CZ wave function instead of the KS. This merits further study. The result shows some dependence on the parameter c, but this dependence is significantly reduced compared to the case of the free form factor [16].

Finally, following [21], we have extracted the effective attenuation cross section $\sigma_{eff}(Q^2)$, which serve as a litmus test of whether "color transparency" has actually been achieved. Our calculations of $\sigma_{eff}(Q^2)$ were done using the same model of correlations and nuclear density as the rest of our calculations.
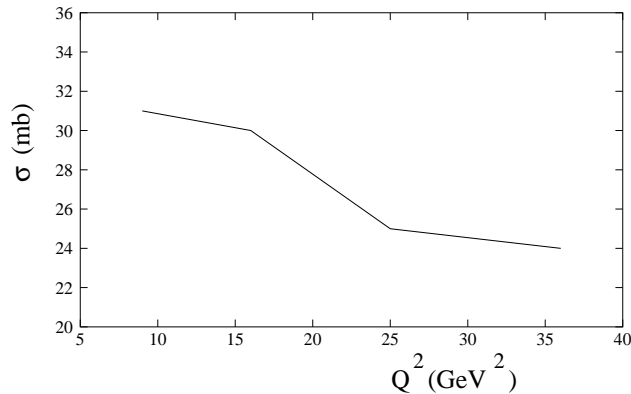
Figure 4: Extracted effective attenuation cross sections $\sigma_{eff}(Q^2)$ as a function of Q^2 exhibit color transparency. The calculations fit the curvature of the A dependence using the same model of nuclear structure and correlations as other calculations. The decrease of $\sigma_{eff}(Q^2)$ with Q^2 is sufficiently large that conventional nuclear physics might be ruled out with sufficiently large Q^2 or sufficiently precise experimental data.

The results (Fig. 4) show a significant decrease of $\sigma_{eff}(Q^2)$ with increasing Q^2 to values well below the Glauber model attenuation cross section, which indicates color transparency.

Acknowledgments

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